Efficient multi-image deconvolution in astronomy

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Abstract

The deconvolution of astronomical images by the Richardson-Lucy method (RLM) is extended here to the problem of multiple image deconvolution and the reduction of boundary effects. We show the multiple image RLM in its accelerated gradient-version SGP (Scaled Gradient Projection). Numerical simulations indicate that the approach can provide excellent results with a considerable reduction of the boundary effects. Also exploiting GPUlib applied to the IDL code, we obtained a remarkable acceleration of up to two orders of magnitude [Prato et al. 2012].

Multiple image deconvolution problem

Multiple image deconvolution problem with Poisson data:

$$\min_{\vec{f}\geq 0} J_0(\vec{f};\vec{g}) = \sum_{j=1}^{p}\sum_{\mathbf{m}\in S}\{\vec{g}_j(\mathbf{m})\ln\frac{\vec{g}_j(\mathbf{m})}{(A_j\vec{f})(\mathbf{m})+\vec{b}_j(\mathbf{m})} + \\ +(A_j\vec{f})(\mathbf{m})+\vec{b}_j(\mathbf{m})-\vec{g}_j(\mathbf{m})\} \quad , \qquad (1)$$

where:

- $\vec{f}$ is the unknown object;
- $\vec{g}_j$ $(j=1,\dots,p)$ are the detected images;
- $A_j\vec{f}=\vec{K}_j * \vec{f}$ $(j=1,\dots,p)$, where $\vec{K}_j$ is the j-th PSF, normalized to unit volume;
- $\vec{b}_j$ $(j=1,\dots,p)$ are the background emissions;
- S is the image domain.

From the standard expectation maximization method [Shepp & Vardi 1982] applied to to this problem, we obtain the *multiple image* RL method

$$\vec{f}^{(k+1)} = \frac{\vec{f}^{(k)}}{p}\sum_{j=1}^{p} A_j^T\frac{\vec{g}_j}{A_j\vec{f}^{(k)}+\vec{b}_j} \quad . \qquad (2)$$

Since

$$\nabla J_0(\vec{f};\vec{g}) = \sum_{j=1}^{p}\left\{\vec{1}-A_j^T\frac{\vec{g}_j}{A_j\vec{f}+\vec{b}_j}\right\} \quad , \qquad (3)$$

algorithm (2) can be seen as a scaled gradient method, with a scaling given, at iteration k, by $\vec{f}^{(k)}/p$. Therefore the application of SGP [Bonettini et al. 2009] to this problem is straightforward.

Boundary effect correction

If the target $\vec{f}$ is not completely contained in the image domain, the previous deconvolution method produce annoying boundary artifacts.

Idea: reconstruct the object $\vec{f}$ over a broader domain $R \supset S$. If we introduce:

- an array $\bar{S}$ containing R and S and such that Fourier transform in $\bar{S}$ can be computed by FFT;
- the masks $\vec{M}_R$, $\vec{M}_S$, defined over $\bar{S}$, which are 1 over R, S respectively and 0 outside;
- the matrices A_j and A_j^T $(j=1,\dots,p)$ defined as

$$(A_j\vec{f})(\mathbf{m}) = \vec{M}_S(\mathbf{m})\sum_{\mathbf{n}\in\bar{S}}\vec{K}_j(\mathbf{m}-\mathbf{n})\vec{M}_R(\mathbf{n})\vec{f}(\mathbf{n}) \qquad (4)$$

$$(A_j^T\vec{g}_j)(\mathbf{n}) = \vec{M}_R(\mathbf{m})\sum_{\mathbf{m}\in\bar{S}}\vec{K}_j(\mathbf{m}-\mathbf{n})\vec{M}_S(\mathbf{m})\vec{g}_j(\mathbf{m}) \ , \qquad (5)$$

where $\vec{K}_j$, $\vec{g}_j$ $(j=1,\dots,p)$ have been extended to $\bar{S}$ by zero padding, then $J_0(\vec{f};\vec{g})$ is given again by (1), with S replaced by $\bar{S}$, while its gradient is now given by

$$\nabla J_0(\vec{f};\vec{g}) = \sum_{j=1}^{p}\left\{A_j^T\vec{1}-A_j^T\frac{\vec{g}_j}{A_j\vec{f}+\vec{b}_j}\right\} \quad . \qquad (6)$$

The domain R can be defined through the functions

$$\vec{\alpha}_j(\mathbf{n}) = (A_j^T\vec{1})(\mathbf{n}) \quad , \quad \vec{n}\in\bar{S} \quad , \qquad (7)$$

$$\vec{\alpha}(\mathbf{n}) = \sum_{j=1}^{p}\vec{\alpha}_j(\mathbf{n}) \quad .$$

in the following way:

$$R = \{\vec{n}\in\bar{S} \mid \vec{\alpha}_j(\vec{n})\geq\sigma; j=1,..,p\} \quad . \qquad (8)$$

where σ is a thresholding value. Then the RL algorithm, with boundary effect correction, is given by

$$\vec{f}^{(k+1)} = \frac{\vec{M}_R}{\vec{\alpha}}\vec{f}^{(k)}\sum_{j=1}^{p}A_j^T\frac{\vec{g}_j}{A_j\vec{f}^{(k)}+\vec{b}_j} \quad , \qquad (9)$$

the quotient being zero in the pixels outside R.

Numerical results

Comparison between:

- Multiple RLM
- SGP

For testing the accuracy of the deconvolution method with boundary effect correction we apply "inverse crime" on an image of nebula NGC7027. The image is partitioned into 4 partially overlapping sub-images, the methods with boundary effect correction are applied and the final reconstruction is obtained as a mosaic of the four partial reconstructions.

Test setting:

- true object: NGC7027 nebula
- blurring: 3 PSF generated according to LINC-NIRVANA [Herbst et al. 2003] model and with equispaced orientations of the baseline (0^o,60^o,120^o)

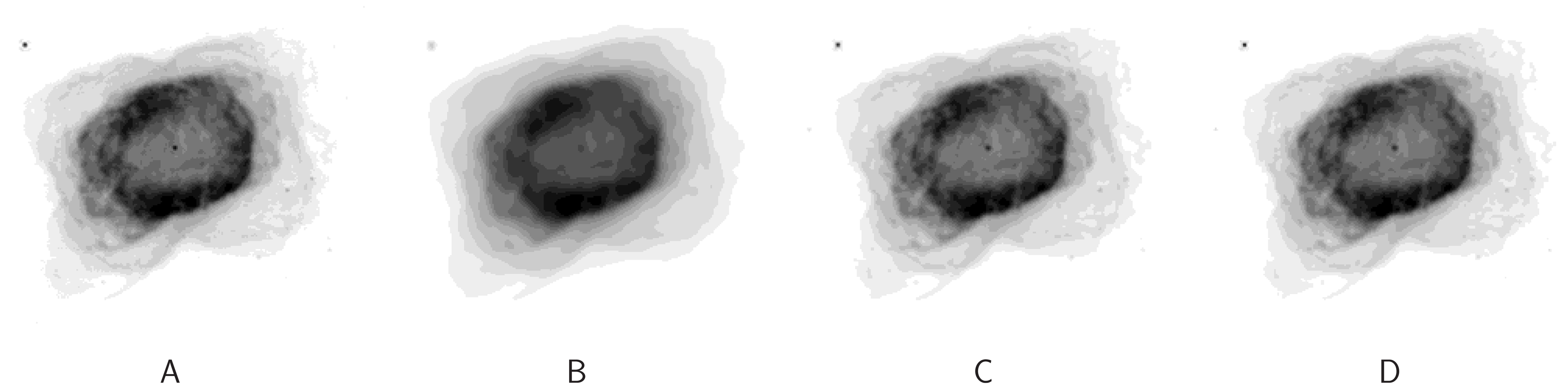

Figure: A: Original Nebula, B: its blurred and noisy image in the case $m=10$ and baseline orientation 0^o; C: reconstruction of the global image; D: reconstruction as a mosaic of four reconstructions of partially overlapping sub-domains, using the algorithms with boundary effect correction.

Table: Reconstruction of the Nebula

Algorithm	It	Err	Sec	SpUp	AlgSpUp
		$m=10$			
RL	2899	0.034	13978	-	-
RL_CUDA	2899	0.034	174.2	80.2	-
SGP	160	0.034	873.3	-	16.0
SGP_CUDA	160	0.034	15.45	56.5	-
		$m=15$			
Algorithm	**It**	**Err**	**Sec**	**SpUp**	**AlgSpUp**
RL	243	0.094	1174	-	-
RL_CUDA	243	0.094	15.28	76.8	-
SGP	11	0.087	69.88	-	16.8
SGP_CUDA	11	0.086	1.532	45.6	-

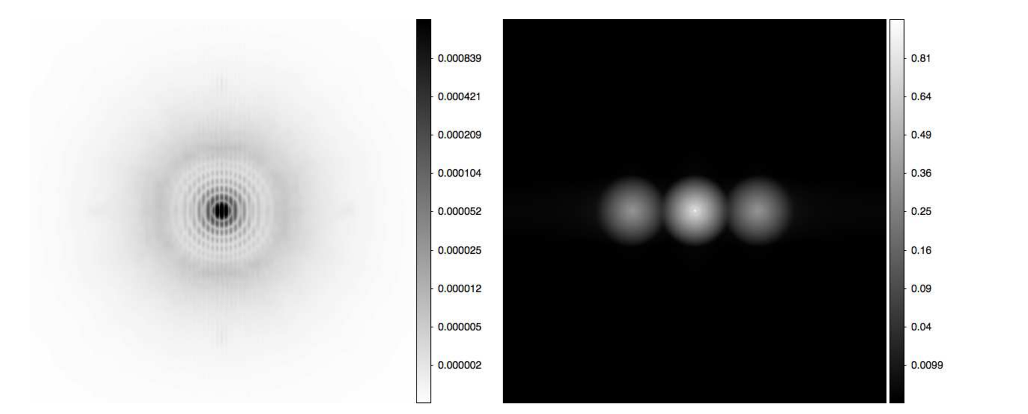

Figure: Simulated PSF of LINC-NIRVANA with SR = 70 % (*left panel*) and corresponding MTF (*right panel*)

References

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