Efficient multi-image deconvolution in astronomy

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Abstract

The deconvolution of astronomical images by the Richardson-Lucy method (RLM) is extended here to the problem of multiple image deconvolution and the reduction of boundary effects. We show the multiple image RLM in its accelerated gradient-version SGP (Scaled Gradient Projection). Numerical simulations indicate that the approach can provide excellent results with a considerable reduction of the boundary effects. Also exploiting GPUlib applied to the IDL code, we obtained a remarkable acceleration of up to two orders of magnitude [Prato et al. 2012].

Multiple image deconvolution problem

Multiple image deconvolution problem with Poisson data:

$$\min_{\mathbf{f}} \left( J(\hat{\mathbf{f}}, \mathbf{g}) \right) = \sum_{j=1}^{p} \left( \frac{\mathbf{M}_0(m) - \mathbf{g}_j(m)}{\mathbf{A}_j \mathbf{f} + \mathbf{b}_j} \right)$$

where:
- $\hat{\mathbf{f}}$ is the unknown object;
- $\mathbf{M}_0$ are the detected images;
- $A_j$ is the $j$-th PSF, normalized to unit volume;
- $\mathbf{b}_j$ are the background emissions;
- $S$ is the image domain.

From the standard expectation maximization method [Shepp & Vardi 1982] applied to this problem, we obtain the multiple image RLM method

$$\hat{\mathbf{f}}^{(k+1)} = \mathbf{f}^{(k)} - \frac{1}{p} \sum_{j=1}^{p} A_j \left( \mathbf{M}_0 - \mathbf{g}_j \right)$$

Since

$$\nabla J(\hat{\mathbf{f}}, \mathbf{g}) = \sum_{j=1}^{p} \left\{ \hat{\mathbf{g}}_j - A_j \hat{\mathbf{f}} \right\}$$

algorithm (2) can be seen as a scaled gradient method, with a scaling given, at iteration $k$, by $\mathbf{f}^{(k)}/p$. Therefore the application of SGP [Bonettini et al. 2009] to this problem is straightforward.

Boundary effect correction

If the target $\hat{\mathbf{f}}$ is not completely contained in the image domain, the previous deconvolution method produce annoying boundary artifacts.

Idea: reconstruct the object $\hat{\mathbf{f}}$ over a broader domain $R \supset S$. If we introduce:
- an array $\bar{S}$ containing $R$ and $S$ and such that Fourier transform in $\bar{S}$ can be computed by FFT;
- the masks $\bar{M}_0$, defined over $\bar{S}$, which are 1 over $R$, $S$ respectively and 0 outside;
- the matrices $A_j$ and $A_j^*$ ($j = 1, \ldots, p$) defined as

$$\begin{align*}
(A_j \hat{\mathbf{f}})(m) &= \bar{M}_0(m) \sum_{m' \in \bar{S}} \bar{K}_j(m - m') \bar{g}_j(m') \\
(A_j^* \hat{\mathbf{f}})(n) &= \bar{M}_0(n) \sum_{n' \in \bar{S}} \bar{K}_j(n - n') \bar{g}_j(n')
\end{align*}$$

where $\bar{K}_j$, $\bar{g}_j$ ($j = 1, \ldots, p$) have been extended to $\bar{S}$ by zero padding, then $J(\hat{\mathbf{f}}, \mathbf{g})$ is given again by (1), with $S$ replaced by $\bar{S}$, while its gradient is now given by

$$\nabla J(\hat{\mathbf{f}}, \mathbf{g}) = \sum_{j=1}^{p} \left\{ \hat{\mathbf{g}}_j^* - A_j^* \hat{\mathbf{f}} \right\}$$

The domain $R$ can be defined through the functions

$$\hat{\mathbf{g}}_j^*(n) = (A_j^* \mathbf{f})^*(n), \quad n \in \bar{S}$$

$$\hat{\mathbf{g}}_j^*(n) = (A_j^* \mathbf{f})^*(n), \quad n \notin \bar{S}$$

in the following way:

$$R = \left\{ n \in \bar{S} \mid \hat{\mathbf{g}}_j^*(n) \geq \sigma, j = 1, \ldots, p \right\}$$

where $\sigma$ is a thresholding value. Then the RL algorithm, with boundary effect correction, is given by

$$\hat{\mathbf{f}}^{(k+1)} = \frac{\bar{M}_0(\mathbf{f})}{\alpha} \sum_{j=1}^{p} A_j^* \left( \hat{\mathbf{g}}_j^* - \hat{\mathbf{f}} \right)$$

the quotient being zero in the pixels outside $R$.

Numerical results

Comparison between:
- Multiple RLM
- SGP

For testing the accuracy of the deconvolution method with boundary effect correction we apply “inverse crime” on an image of nebula NGC7027. The image is partitioned into 4 partially overlapping sub-images, the methods with boundary effect correction are applied and the final reconstruction is obtained as a mosaic of the four partial reconstructions.

Test setting:
- true object: NGC7027 nebula
- blurring: 3 PSF generated according to LINC-NIRVANA [Herbst et al. 2003] model and with equispaced orientations of the baseline (0°,60°,120°)

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Table: Reconstruction of the Nebula

$\alpha = 10$

Figure: A: Original Nebula, B: its blurred and noisy image in the case $m = 10$ and baseline orientation 0°; C: reconstruction of the global image; D: reconstruction as a mosaic of four reconstructions of partially overlapping sub-domains, using the algorithms with boundary effect correction.

Figure: Simulated PSF of LINC-NIRVANA with SR = 70 % (left panel) and corresponding MTF (right panel)

References