Semi-blind deconvolution for Fourier-based image restoration

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Abstract

In this work we develop a new optimization algorithm for image reconstruction problems from Fourier data with uncertanties on the spatial frequencies corresponding to the measured data. By considering such dependency on the frequencies as a further unknown, we obtain a so-called semi-blind deconvolution. Both the image and the spatial frequencies are obtained as solutions of a reformulated constrained optimization problem, approached by an alternating scheme. Numerical tests on simulated data, based on the imaging hardware of the NASA RHESSI satellite, show that the proposed approach provides some improvements in the reconstruction.

Mathematical formulation

Fourier-based image restoration can be modeled as an inverse problem

$$\mathbf{g} = \mathcal{A}(\boldsymbol{\omega})\mathbf{f} + \boldsymbol{\eta}$$

where

- $\mathbf{g} \in \mathbb{C}^N$ is the available complex data (called *visibilities*)
- $\mathbf{f} \in \mathbb{R}^{n^2}$ is the unknown true image
- $\boldsymbol{\omega} = (\mathbf{u}, \mathbf{v}) \in \mathbb{R}^N$ are the unknown spatial frequencies
- $\mathcal{A}$ is the discrete Fourier trasform,

$$(\mathcal{A}(\mathbf{u},\mathbf{v})\mathbf{f})_k = \sum_{j,h=1}^{n} \mathbf{f}_{jh} \exp\left(2\pi i (x_j u_k + y_h v_k)\right), \quad k = 1, \dots, N$$

- $\boldsymbol{\eta}$ models the noise

The vector of unknowns $(\mathbf{f}, \boldsymbol{\omega})$ can be obtained as the solutions of the constrained optimization problem [2]

$$\min_{\substack{\mathbf{f}\geq 0 \\ \boldsymbol{\omega}^l \leq \boldsymbol{\omega} \leq \boldsymbol{\omega}^u}} \mathcal{F}(\mathbf{f}, \boldsymbol{\omega}) = \frac{1}{2}\|\mathcal{A}(\boldsymbol{\omega})\mathbf{f} - \mathbf{g}\|^2_{\mathbb{C}^N}. \qquad (1)$$

Alternating Optimization

Alternating Optimization exploits the separability of the unknowns $(\mathbf{f}, \boldsymbol{\omega})$ and generates an iterative sequence $\{(\mathbf{f}^{(h)}, \boldsymbol{\omega}^{(h)})\}$,

$$\mathbf{f}^{(h)} = \operatorname*{argmin}_{\mathbf{f}\geq 0} \|\mathcal{A}(\boldsymbol{\omega}^{(h-1)})\mathbf{f} - \mathbf{g}\|^2_{\mathbb{C}^N} \qquad (2)$$

$$\boldsymbol{\omega}^{(h)} = \operatorname*{argmin}_{\boldsymbol{\omega}^l \leq \boldsymbol{\omega} \leq \boldsymbol{\omega}^u} \|\mathcal{A}(\boldsymbol{\omega})\mathbf{f}^{(h)} - \mathbf{g}\|^2_{\mathbb{C}^N} \qquad (3)$$

that convergences to a stationary point for problem (1).
The subproblems (2) and (3) can be solved **inexactly** [3] through a suitable descent method, such as Scaled Gradient Projection (SGP) [1].

Alternating Optimization scheme

Choose $\mathbf{f}^{(0)} \geq 0$, $\boldsymbol{\omega}^l \leq \boldsymbol{\omega}^{(0)} \leq \boldsymbol{\omega}^u$

FOR $h = 1, 2, \dots$ DO:

STEP $\mathbf{f}$: Compute $\mathbf{f}^{(h)}$ by applying SGP to (2) starting from $\mathbf{f}^{(h-1)}$.

STEP $\boldsymbol{\omega}$: Compute $\boldsymbol{\omega}^{(h)}$ by applying SGP to (3) starting from $\boldsymbol{\omega}^{(h-1)}$.

END

Numerical results: the RHESSI mission

The solar satellite RHESSI [5] has been launched by NASA on February 5 2002 with the aim of providing new insights for the comprehension of the acceleration mechanisms occurring during solar flares. RHESSI encodes spatial information through the temporal modulation of photon flux by a set of nine rotating collimators [4]. These data are rather straightforwardly converted into *visibilities*, that are 2D spatial Fourier components corresponding to the spatial frequencies $\boldsymbol{\omega} = (\mathbf{u}, \mathbf{v})$ lying on nine concentric circles. Both the X-ray image and the spatial frequencies are unknown.

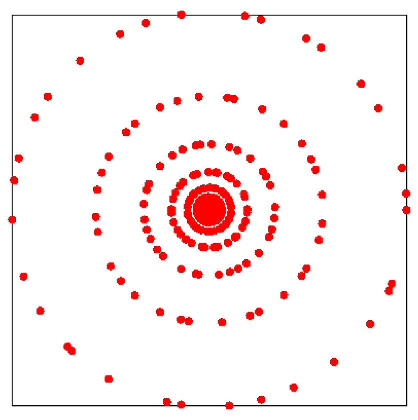

Figure: Example of a typical sampling of RHESSI data in the frequency plane

$\boldsymbol{\omega} = (\mathbf{u}, \mathbf{v})$ can be written in polar coordinates. The radial component can be determined through physical properties of the detector, while the angular component is actually unknown.

Comparison between:

- Deconvolution with $\boldsymbol{\omega}$ set equal to $\frac{1}{2}(\boldsymbol{\omega}^l + \boldsymbol{\omega}^u)$ (**SpaceD**) [2]
- Semi-blind deconvolution: (**GP-GP**)
  - subproblem (2): SGP, scaling matrix $\mathbf{D} = \mathbf{I}$
  - subproblem (3): SGP, scaling matrix $\mathbf{D} = \mathbf{I}$
- Semi-blind deconvolution: (**GP-GN**)
  - subproblem (2): SGP, scaling matrix $\mathbf{D} = \mathbf{I}$
  - subproblem (3): SGP, scaling matrix $\mathbf{D} = \mathbf{J}^T\mathbf{J}$, $\mathbf{J}$ Jacobian matrix of $\mathcal{F}$ wrt $\boldsymbol{\omega}$

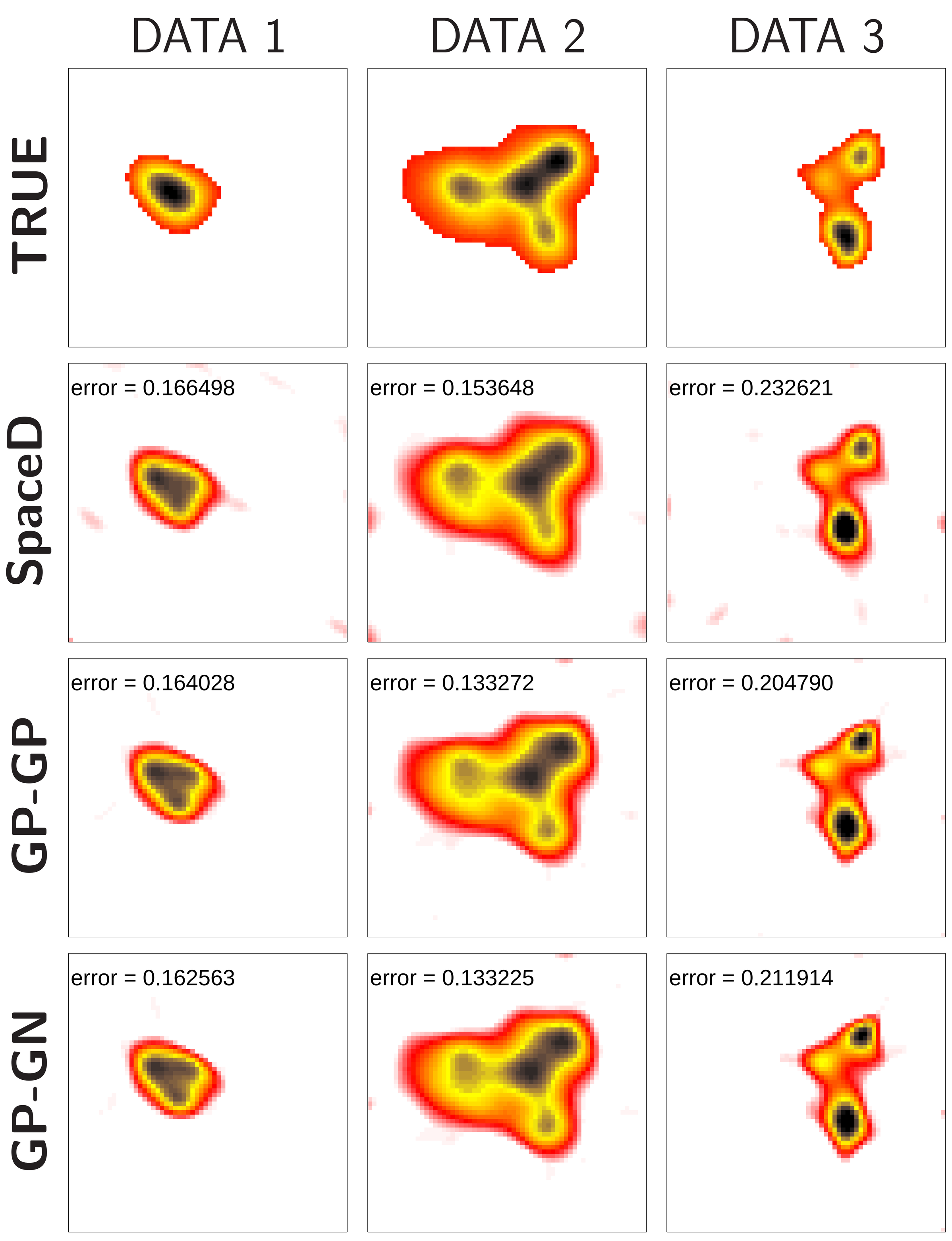

Figure: Images reconstructions

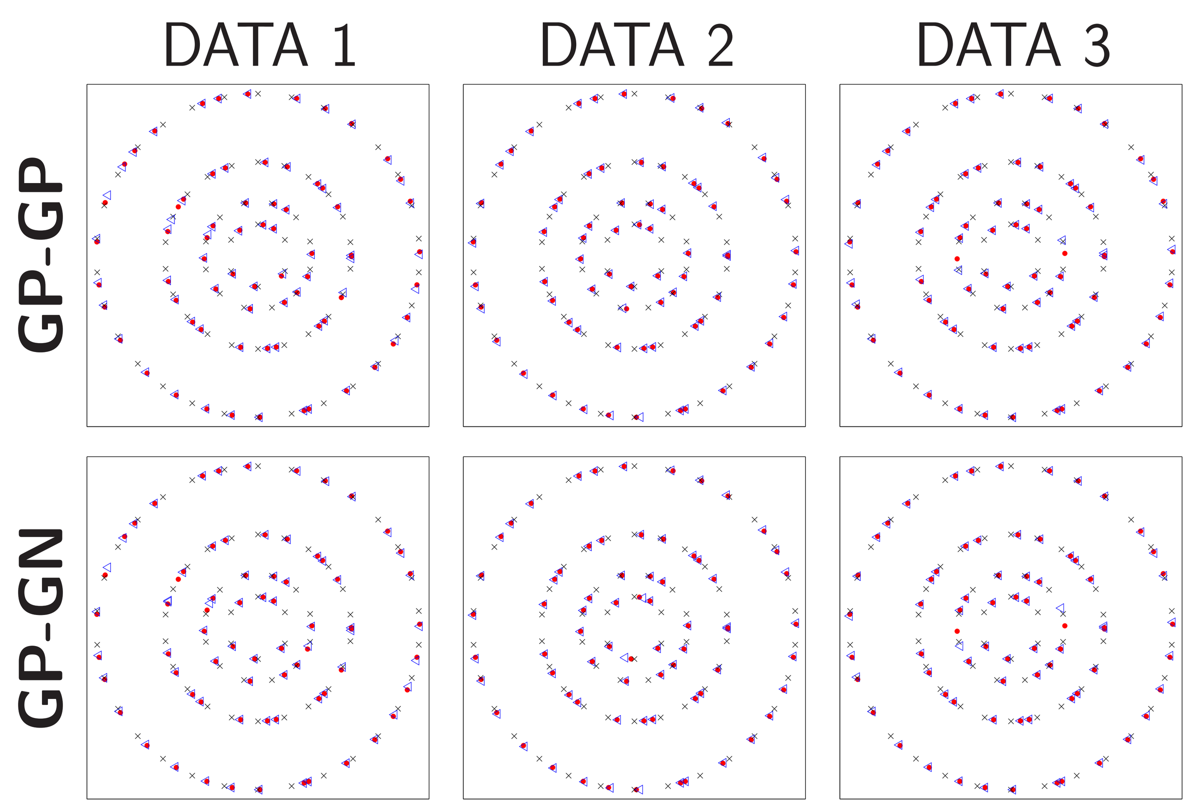

Figure: Frequencies reconstructions: plot of $\mathbf{u}$ versus $\mathbf{v}$ (4 of the 9 circles). Red points are the true values; black crosses are the values fixed as for SpaceD; blue triangles are the reconstructed values.

	DATA 1	DATA 2	DATA 3
SpaceD	0.025931	0.025931	0.025931
GP-GP	0.006005	0.004037	0.007707
GP-GN	0.007331	0.005895	0.009926

Table: Relative errors on $\boldsymbol{\omega}$

References

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