An alternating minimization method for blind deconvolution in astronomy

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Abstract

Blind deconvolution is the problem of image deblurring when both the original object and the blur are unknown. In this work, we show a particular astronomical imaging problem, in which p images of the same astronomical object are acquired and convolved with p different Point Spread Functions (PSFs). According to the maximum likelihood approach, this becomes a constrained minimization problem with p+1 blocks of variables, whose objective function is globally non convex. Thanks to the separable structure of the constraints, the problem can be treated by means of an inexact alternating minimization method whose limit points are stationary for the function [1]. This method has been tested on some realistic datasets and the numerical results are hereby reported to show its effectiveness on both sparse and diffuse astronomical objects.

Imaging with Large Binocular Telescope

The Fizeau interferometer LINC-NIRVANA [2] of the Large Binocular Telescope (LBT) detects p different images of the same object, corresponding to p different rotations of LBT, in order to get the maximum resolution in all directions.

Each observed image is a vector $\boldsymbol{y}_j \in \mathbb{R}^n$, $\boldsymbol{y}_j \sim Poisson(\boldsymbol{K}_j * \boldsymbol{x} + \boldsymbol{b}_j)$ where $j = 1, \dots, p$ and:

- $\boldsymbol{x} \in \mathbb{R}^n$ is the unknown object
- $\boldsymbol{K}_j \in \mathbb{R}^n$ is the $j-$th Point Spread Function (PSF)
- $\boldsymbol{b}_j \in \mathbb{R}^n$ is the $j-$th constant background term.

Problem: given $(\boldsymbol{y}_j, \boldsymbol{b}_j)$, $j = 1, \dots, p$, find an estimate of the true object $\boldsymbol{x}$ and the PSFs $\boldsymbol{K}_1, \ \boldsymbol{K}_2 \dots, \boldsymbol{K}_p$

Maximum likelihood approach: minimize the negative logarithm of the likelihood function, i.e.

$$\min_{\boldsymbol{x}\in\Omega,\ \boldsymbol{K}_j\in\Omega_j} J_0(\boldsymbol{x}, \boldsymbol{K}_1, \dots, \boldsymbol{K}_p; \boldsymbol{y}, \boldsymbol{b}) = \sum_{j=1}^{p} D_{KL}(\boldsymbol{y}_j; \boldsymbol{K}_j * \boldsymbol{x} + \boldsymbol{b}_j) \quad (1)$$

- $D_{KL}(\boldsymbol{y}_j; \boldsymbol{K}_j * \boldsymbol{x} + \boldsymbol{b}_j) =$

$$= \sum_{i=1}^{n} \left\{ \boldsymbol{y}_j(i) \ln \frac{\boldsymbol{y}_j(i)}{(\boldsymbol{K}_j * \boldsymbol{x})(i) + \boldsymbol{b}_j(i)} + (\boldsymbol{K}_j * \boldsymbol{x})(i) + \boldsymbol{b}_j(i) - \boldsymbol{y}_j(i) \right\}$$

is the generalized Kullback-Leibler divergence

- $\Omega = \{\boldsymbol{x} \in \mathbb{R}^n \ : \ \boldsymbol{x} \geq 0, \ \sum_{\ell=1}^{n} x(\ell) = c\}$

$\Omega_j = \{\boldsymbol{K} \in \mathbb{R}^n \ : \ 0 \leq \boldsymbol{K} \leq s_j, \ \sum_{\ell=1}^{n} \boldsymbol{K}(\ell) = 1\}$

are the closed, convex, separable constraints, where s_j is an upper bound derived from the Strehl ratio of LBT and c is the average flux of the p detected images (after background subtraction).

The Cyclic Block Gradient Projection algorithm

Choose the initial guesses $\boldsymbol{x}^{(0)}, \boldsymbol{K}_1^{(0)}, \dots, \boldsymbol{K}_p^{(0)}$ and $p+1$ positive integers $L_0, L_1, \dots, L_p$.

FOR $k = 0, 1, 2, ...$ the $(k+1)$-th iterates are computed as follows:

1. Start from $\boldsymbol{x}^{(k)}$ and compute $\boldsymbol{x}^{(k+1)}$ with $L_0^{(k)} \leq L_0$ SGP iterations applied to

$$\min_{\boldsymbol{x}\in\Omega} J_0(\boldsymbol{x}, \boldsymbol{K}_1^{(k)}, \dots, \boldsymbol{K}_p^{(k)}; \boldsymbol{y}, \boldsymbol{b})$$

2. For $j = 1, \dots, p$, start from $\boldsymbol{K}_j^{(k)}$ and compute $\boldsymbol{K}_j^{(k+1)}$ with $L_j^{(k)} \leq L_j$ SGP iterations applied to

$$\min_{\boldsymbol{K}\in\Omega_j} J_0(\boldsymbol{x}^{(k+1)}, \boldsymbol{K}_1^{(k+1)}, \dots, \boldsymbol{K}_{j-1}^{(k+1)}, \boldsymbol{K}, \boldsymbol{K}_{j+1}^{(k)}, \dots, \boldsymbol{K}_p^{(k)}; \boldsymbol{y}, \boldsymbol{b})$$

END

At each step k, we solve **inexactly** $p+1$ subproblems of the form

$$\min_{\boldsymbol{z}\in\Omega} J(\boldsymbol{z})$$

with $\Omega \subset \mathbb{R}^n$ closed and convex and $J \in C^1(\Omega)$, by means of the **scaled gradient projection (SGP) method** [3], which generates a sequence

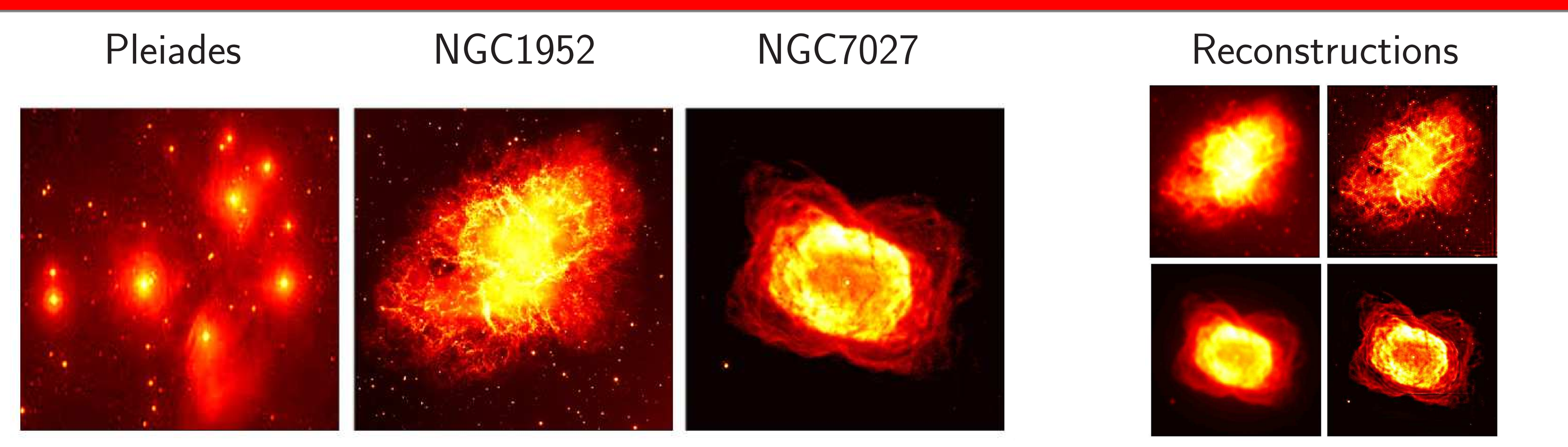
$$\boldsymbol{z}^{(k+1)} = \boldsymbol{z}^{(k)} + \lambda_k \boldsymbol{d}^{(k)}$$

- $\boldsymbol{d}^{(k)} = P_{\Omega, D_k^{-1}}(\boldsymbol{z}^{(k)} - \alpha_k D_k \nabla J(\boldsymbol{z}^{(k)})) - \boldsymbol{z}^{(k)}$ is the feasible descent direction, where $\alpha_k \in [\alpha_{min}, \alpha_{max}]$ is a scalar step-length parameter, D_k is a positive definite matrix whose eigenvalues are bounded from above and below by two constants independent of k and $P_{\Omega, D_k^{-1}}$ is the projection onto Ω associated with the norm induced by D_k^{-1}
- $\lambda_k = \theta^m$, with $\theta \in (0,1)$ and m being the smallest integer such that an Armijo-like successive stepsize reduction rule is satisfied for λ_k.

Every limit point of this sequence is stationary for problem (1).

Numerical results

- **Initial guesses:** $\boldsymbol{x}^{(0)} \geq 0$ is a constant image with the average flux of the background-subtracted images, while $\boldsymbol{K}_j^{(0)}$ is the autocorrelation of the corresponding ideal PSF
- **Inner iterations:** 50 SGP iterations on the object and 1 on each PSF for the Pleiades. No similar rule for the diffuse objects
- **Scaling matrix:** $D_k = \text{diag}(\min(C_2, \max(C_1, \boldsymbol{z}^{(k)})))$, where C_1 and C_2 are positive constants
- **Step-length:** alternation of the two Barzilai-Borwein rules
- **Results:** For the Pleiades, the method provides an excellent reconstruction of both the object and the PSFs.
  For the two nebulas, there are sensible improvements with respect to the single image blind deconvolution approach [4].



Images	$RMSE^{obj}$	$RMSE_1^{obj}$	$RMSE_2^{obj}$	$RMSE_1^{psf}$	$RMSE_2^{psf}$
Pleiades	0.18%	33.19%	1.89%	44.48%	1.98%
NGC1952	12.43%	15.78%	15.17%	44.48%	30.33%
NGC7027	4.8%	14.89%	11.54%	44.48%	32.45%

$RMSE^{obj}$ ($RMSE_1^{obj}$): SGP with true (initial) PSF; $RMSE_1^{psf}$: true PSF vs initial PSF
$RMSE_2^{obj}$: CBGP (best error); $RMSE_2^{psf}$: true PSF vs restored PSF

[1] Bonettini S. 2011, Inexact block coordinate descent methods with application to non-negative matrix factorization, IMA J. Numer. Anal. 31, 1431-1452.

[2] Herbst T.M. et al 2003, LINC-NIRVANA: a Fizeau beam combiner for the Large Binocular Telescope, Proc. SPIE 4838, 456-465.

[3] Bonettini S., Zanella R. and Zanni L. 2009, A scaled gradient projection method for constrained image deblurring, Inverse Probl. 25, 015002.

[4] Prato M., La Camera A., Bonettini S. and Bertero M. 2013, A convergent blind deconvolution method for post-adaptive-optics astronomical imaging, Inverse Probl. 29, 065017.