Motivations

Health hazards for patients due to ionizing radiation in Computed tomography (CT) can be reduced by limiting the irradiation to a subregion of the object to be reconstructed, the so-called region-of-interest (ROI) [1].

Goal

Obtaining a stable reconstruction of the ROI without any assumption on the size and location of the ROI and overcoming the ill-posedness of the problem and the instability of naive local reconstruction algorithms.

State of the art

Examples of recoverable regions:
(a) from (at least) one projection view the object is completely recovered;
(b) a known subregion inside the ROI is given;
(c) no assumption on the size and location of the ROI, except for its convex shape.

2D discrete setting

Denoting with \( W \) the \( K P \times N^2 \) forward projection matrix, the data fidelity and consistency equations read as follows:

\[
\text{MWf} = \text{My} = y_0 \quad \text{(data fidelity)}
\]

\[
(\text{IKP} - \text{M})Wf = (\text{IKP} - \text{M})y \quad \text{(data consistency)}
\]

where \( K \) is the number of projection angles, \( P \) is the number of detector elements, \( N \) is the width in pixel of the reconstructed object.

Unfortunately, these equations alone do not lead to a unique solution [3]. A suitable one can be derived using a Tikhonov-like regularization:

\[
\min_{f \in S} \frac{1}{2} \| f - f_0 \|^2 + \frac{\lambda}{2} \| \Phi (f) - y_0 \|^2
\]

where \( \Phi(f) = f + \rho (\text{TV}(f) / f) \) and \( \Phi \) is the shearlet (resp. wavelet) transform [4].

Slight modifications of the objective function can be taken into account, coupling the regularization term with, for instance, a Total Variation term:

\[
\min_{f \in S} \frac{1}{2} \| f - f_0 \|^2 + \rho (\text{TV}(f) / f)
\]

Here, \( \lambda \) and \( \rho \) are regularization parameters, \( \delta \) is the TV smoothing parameter.

Distance-Driven method

Each object pixel (voxel) and detector cell is mapped onto a common axis (plane) by its projected boundary midpoints.

\[
a_h = \frac{\xi_h - \xi_{h-1}}{2}, \quad a_w = \frac{\eta_w - \eta_{w-1}}{2}, \quad a_{h,w} = \frac{\xi_h - \xi_{h-1}}{2} \frac{\eta_w - \eta_{w-1}}{2}
\]

The length of the overlap is used as projection weight [5].

2D problem setting

The aim of ROI CT is to reconstruct an integrable function \( f \) from its Radon projections \( y_0 \) known only within a subregion inside the field of view, while the rest of the image is ignored. This is accomplished by setting:

\[
y_0(\theta, \tau) = M(\theta, \tau) R f(\theta, \tau)
\]

where \( R f(\theta, \tau) = \int \delta(\tau - x \cdot e_\theta) f(x) \, dx \) is the Radon transform of \( f \) at \( (\theta, \tau) \) and the mask \( M(\theta, \tau) = 1 \) on \( (\theta, \tau) \) identifies the ROI \( S \) in the sinogram space [2].

Given \( y_0 \) defined on \( P(S) \), the goal is to extrapolate it to the region outside \( P(S) \), ensuring that the Radon projections \( y = R f \) comes from the Radon transform of a function \( f \) in \( L^1 \cap L^2 \):

\[
M R f = y_0 \quad \text{(data fidelity)}
\]

\[
(1 - M) R f = (1 - M)y_0 \quad \text{(data consistency)}
\]

Future perspectives

- Investigate sparse reconstruction
- Obtain stable reconstructions from (Poisson) noisy sinograms
- Separable footprint method for system matrix
- Apply the same machinery to helical CT

References


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