Total Variation regularization algorithms for tomosynthesis imaging

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Abstract

This work addresses the problem of 3d imaging reconstruction from Digital Breast Tomosynthesis projections. DBT is modeled as an unconstrained minimization problem, where the objective function is a convex function of the gradient of the unknown image (Total Variation operator) plus the Least Squares data fidelity term: numerical experiments on a breast phantom widely used in tomographic simulations show the efficiency of this model for DBT. A Scaled Gradient Projection algorithm is compared to a Fixed Point one: both of them achieve very accurate results at their convergence, but the FP method also provides good quality reconstructions in its early stopping outputs.

Tomosynthesis and problem formulation

The DBT technique is characterized by few 3d scans, performed by an X-ray source that wheels in a limited plane angular range. Each scan provides the 2d projection of a compressed breast, onto a digital flat detector, thanks to a low-dose X-ray cone beam. The breast 3d digital reconstruction is visually conceived as a stack of many 2d layers, overlaying in the vertical z-direction. Due to the incompleteness of the projection data, DBT image reconstruction is a challenging inverse problem.

Figure: DBT machinery.

The image formation process is modelled as an underdetermined linear system $Mf = g$ and the reconstructed image f is the solution of:

$$\min_{f \in \mathbb{R}^N} J(f) = \frac{1}{2}\|Mf - g\|_2^2 + \lambda TV(f)$$

- f is the breast volume discretized in $N := N_x \times N_y \times N_z$ voxels;
- g is the collection of all the projections of f on a flat detector (made of $n_x \times n_y$ pixels), from n_θ angles taken in a $[-\phi, \phi]$ interval ($n := n_x \times n_y \times n_\theta$ and $n \ll N$);
- M is a matrix of size n×N, representing how the projector works on the detector;
- $TV(f) := \sum_{j=1}^{N} \|D_j f + \gamma\|_2$ is the differentiable discrete Total Variation of f (with $D_j f \in \mathbb{R}^3$ 3d-discrete gradient in f_j) and $\lambda > 0$ is the regularization parameter.

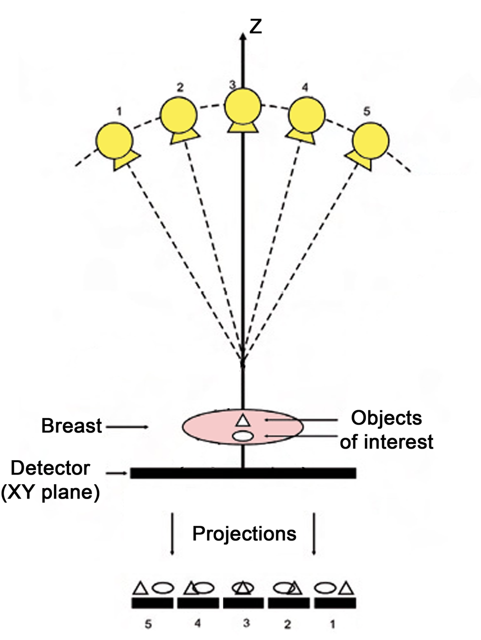

Figure: DBT system scheme in yz plane.

Scaled Gradient Projection algorithm [1]

The constrained problem $\min_{f \in \Omega} J(f)$ on $\Omega = \{f \geq 0\}$ is solved with:

SGP algorithm

choose initial f_0 and set k=0, $\theta \in (0,1)$, *and* $0 < \alpha_{min} < \alpha_{max}$
while (convergence)

1. choose $\alpha_k \in [\alpha_{min}, \alpha_{max}]$ with BB rules and the scaling diagonal matrix D_k
2. compute the projection $h_k = P_{\Omega, D_k^{-1}}(f_k - \alpha_k D_k \nabla J(f_k))$
3. if $h_k = f_k$ then stop
4. compute the descent direction $d_k = h_k - f_k$
5. execute a backtracking loop to get the steplength μ_k
6. update $f_{k+1} = f_k + \mu_k d_k$ and set k = k+1

end

[1] R. Zanella et al., *Efficient gradient projection methods for edge-preserving removal of Poisson noise, Inverse Problems 25 (2009)*

Fixed Point algorithm [2]

Considering $L_k = L(f_k)$ where $L(f)$ is an operator such that $L(f_k)f_k = \nabla TV(f_k)$, the optimization problem is solved with:

FP algorithm

choose initial f_0 and set k=0
while (convergence)

1. compute the gradient $g_k = \nabla J(f_k)$
2. compute the approximated hessian $H_k = M^t M + \lambda L(f_k)$
3. solve the linear system $H_k s_k = -g_k$ with CG method
4. update the approximate solution $f_{k+1} = f_k + s_k$
5. set k=k+1

end

[2] C. Vogel, *Computational methods for Inverse Problems*, SIAM 2002.

Numerical results

The test problem is defined computing the projections $g = Mf^* + \eta$, where f^* is the true volume and η is a gaussian noise of $rnl = \frac{\|\eta\|}{\|g\|} = 10^{-3}$.

Geometry parameters:

- $N_x = N_y = 128$, $N_z = 15$
- $n_x = n_y = 128$, $n_\theta = 13$
- Angular range: $[-17, 17]$
- n=212992, N=245760

Algorithm parameters:

- $\gamma = 10^{-3}$
- $\lambda = 0.01$
- maxit_cg_fp = 100

Evaluation criteria:

- relative error $RE(f) = \frac{\|f - f^*\|}{\|f^*\|}$
- mean and standard-deviation values inside a uniform ROI (background value = 0.17)
- 20 second runtime solutions and convergence outputs

Stopping criterion:

- $abs(\frac{J_k - J_{k-1}}{J_{k-1}}) < 10^{-6}$

method	RE	iterations	J(f)	mean	std-dev
SGP in 20 secs	0.2069	22	337.17	0.1725	1.8922 e-3
FP in 20 secs	0.1499	5 (+417)	51.12	0.1702	8.9327 e-4
SGP up to conv.	0.0673	1513	39.74	0.1704	4.2243 e-4
FP up to conv.	0.0655	100 (+9917)	44.33	0.1702	4.5559 e-4

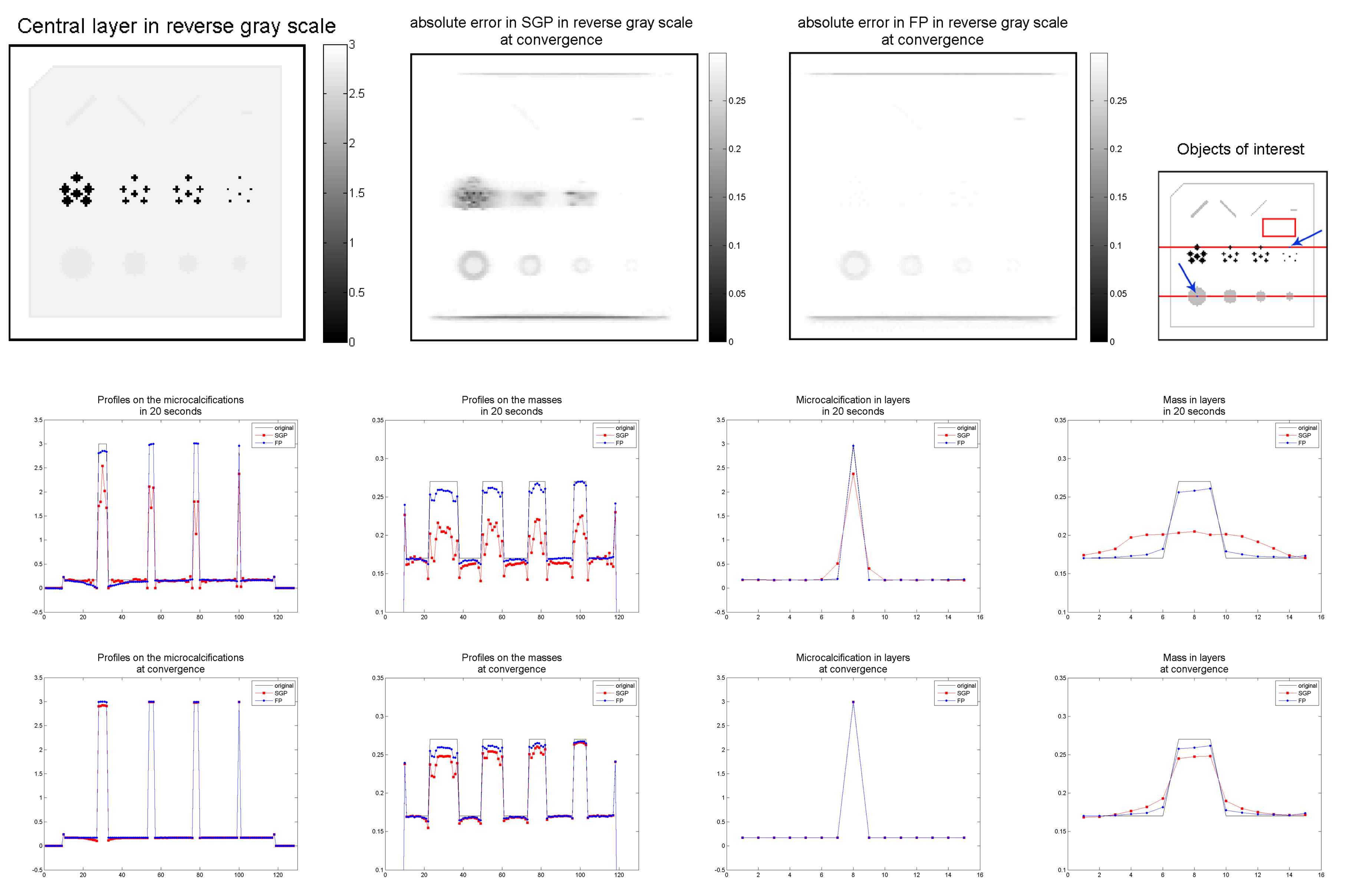

Figure: Central layer of the digital mammographic phantom *CIRS mod. 015*, in the exact version and as the absolute error in the convergence outputs. Profiles of interest on the 20 second and convergence outputs are shown, for the SGP and FP algorithms.