Reconstruction of solar flare images using interpolated visibilities

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methods for image and data analysis
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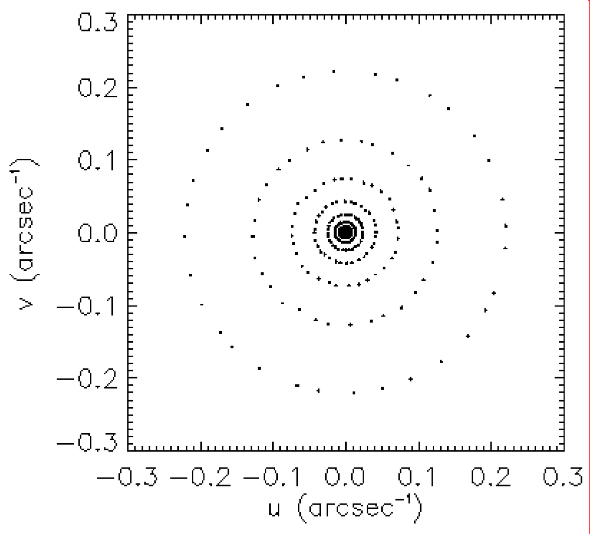
Objective

One possibility to create images of high energy X-rays and γ-rays is the use of a set of Rotational Modulation Collimators (RMCs). The combined effect of the collimators' grids and the hardware rotation is a set of spatial Fourier components, called visibilities, sampled on spatial frequencies distributed over concentric circles. We introduce a fast and reliable method for X-ray imaging by applying an inverse FFT code to interpolated visibilities. We also show that super-resolution effects can be obtained by utilizing a projected iterative algorithm.

RHESSI and visibilities

The Reuven Ramaty High Energy Solar Spectroscopic Imager (*RHESSI*) [1], launched by NASA on February 2002, produces images with the finest angular and spectral resolution ever achieved at hard X-ray and γ-ray energies. Such **imaging spectroscopy** provides a powerful tool with which to explore the underlying physics of particle acceleration and transport in solar flares.

RHESSI encodes spatial information through the temporal modulation of photon flux by a set of nine Rotating Modulation Collimators (RMCs) [2]. This information is rather straightforwardly converted to photon **visibilities**, which are 2D spatial Fourier components corresponding to spatial frequencies (u,v) lying on nine concentric circles.



Visibility-based imaging methods

$$V(u,v) := (\mathfrak{F}I)(u,v) = \iint I(x,y)e^{2\pi i(ux+vy)}dxdy$$

visibility **Fourier Transform** **photon flux**

Visibilities advantages:

- full calibration: no remaining instrument dependence
- well determined statistical errors (visibilities are linear combinations of measured counts)
- background automatically removed
- indication of systematic errors provided by redundancy
- Fourier-based imaging possible, e.g.:
  - **back-projection** (direct Fourier inversion of the measured visibilities). Drawbacks: significant sidelobes, limited usefulness;
  - **Maximum Entropy Method (MEM)** [3]. Drawbacks: application not always successful in these circumstances;
  - **Forward Fit** (best fit of parametrized simple functional forms). Drawbacks: applicability limited to sources whose morphology matches predetermined functional form.

uv - smooth

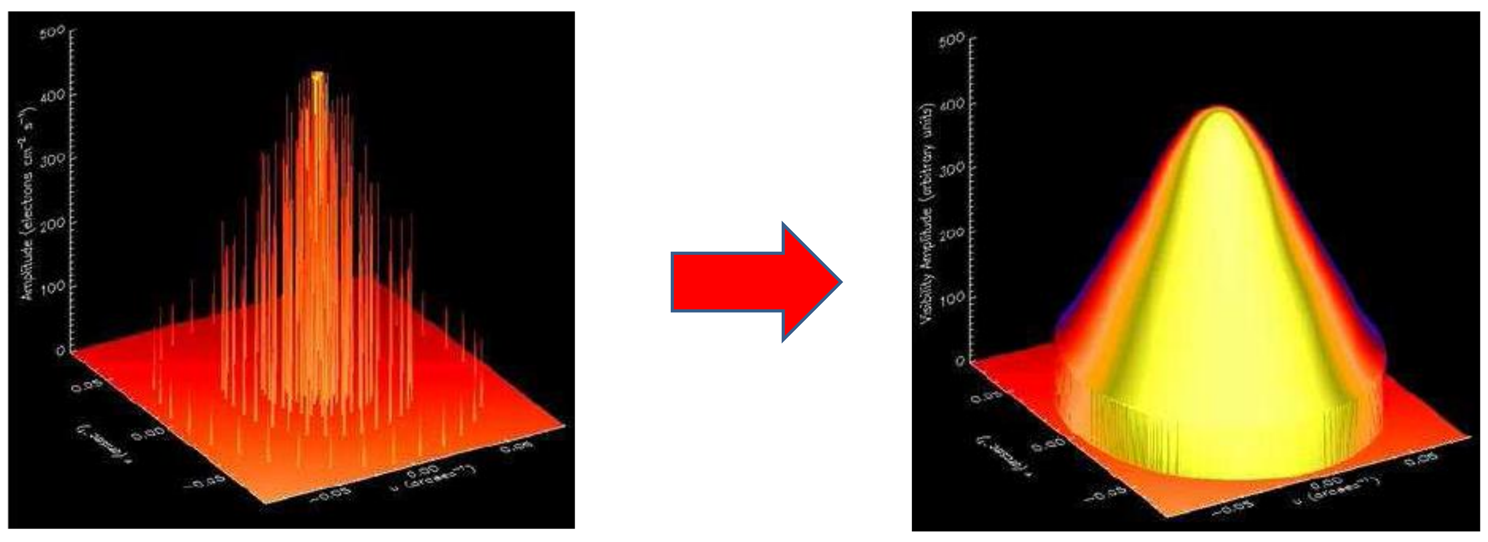
Here we fill a need by introducing a robust, widely-applicable algorithm for reconstructing RHESSI visibility-based data. It proceeds by smoothing the observed visibilities in the spatial frequency plane prior to Fourier inversion. For this reason we called this method **uv – smooth** [4].
The algorithm consists of a two-step process: 1. **interpolation** to generate a smooth continuum of visibilities within the disk in the (u,v) plane spanned by the available data; 2. the imposition of image **positivity** through a Fast Fourier Transform (FFT)-based iterative method.

Interpolation

Idea: use the 2D FFT algorithm to get the image in a fast and natural way.
Problem: if the data are sampled sparsely and not uniformly in the (u,v) plane, as occurs with *RHESSI* visibilities, 2D FFT is not applicable.
Solution: interpolate and re-sample the visibility set.
Improvements:

- information also for spatial frequancies **inside** the nine circles (which, in principle, corresponds to "virtual" subcollimators with angular resolution between the minimum and the maximum values available with *RHESSI*'s hardware) are achieved;
- with the new (uniform) re-sampling on the visibilities in the (u,v) plane, the 2D FFT routine can be applied.

The interpolation step is performed through a thin-plate spline algorithm.



FFT + positivity constraint

Idea: extract information also for frequencies **outside** the nine circles to get a super-resolution effect of the reconstructed images.
Problem: find the function I such that

$$V(u,v) = \chi_B(u,v)(\mathfrak{F}I)(u,v)$$

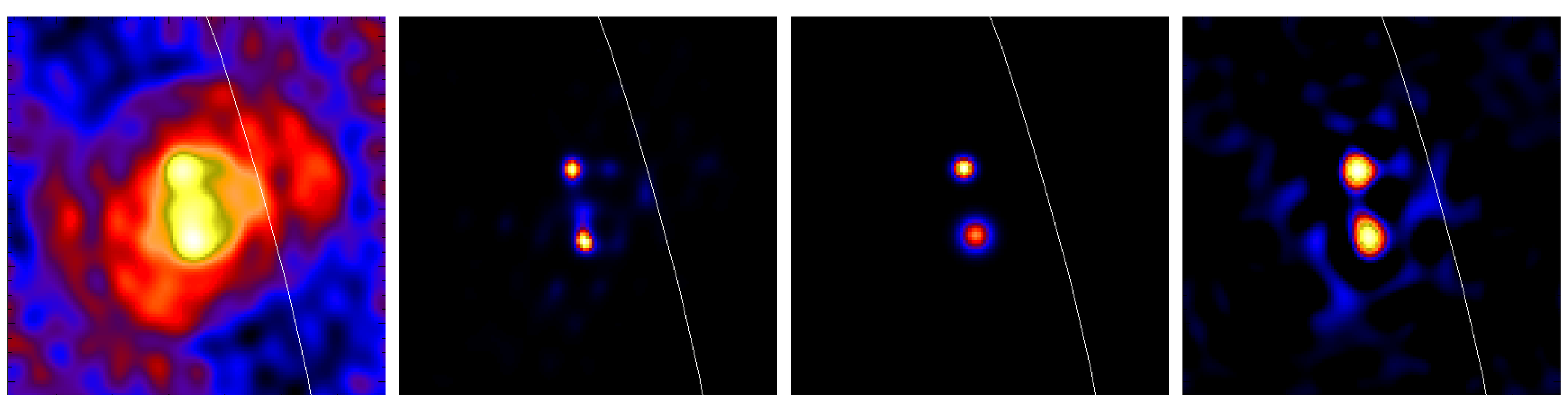
where B is the band in which RHESSI provides the visibilities and χ_B is the characteristic function of B.
Solution: Gerchberg-Papoulis method [5,6] with positivity constraint:

1. put $I^{(0)}$ equal to the null map;
2. for k=0,1,…
   (a) calculate the Fourier Transform $\mathfrak{F}I^{(k)}$ of $I^{(k)}$;
   (b) calculate
   $$\mathfrak{F}I^{(k+1)}(u,v) = \mathfrak{F}I^{(k+1)}(u,v) + \tau(V(u,v) - \chi_B(u,v)(\mathfrak{F}I)(u,v))$$
   (c) calculate the Inverse Fourier Transform $I^{(k+1)}$ of $\mathfrak{F}I^{(k+1)}$;
   (d) project $I^{(k+1)}$ on the set of the real positive numbers;
   (e) stopping rule: if satisfied, $I^{(k+1)}$ is the desired approximation. Else, go back to step 2.

The steplength parameter τ has to be properly choosen in order to assure the convergence of the algorithm.

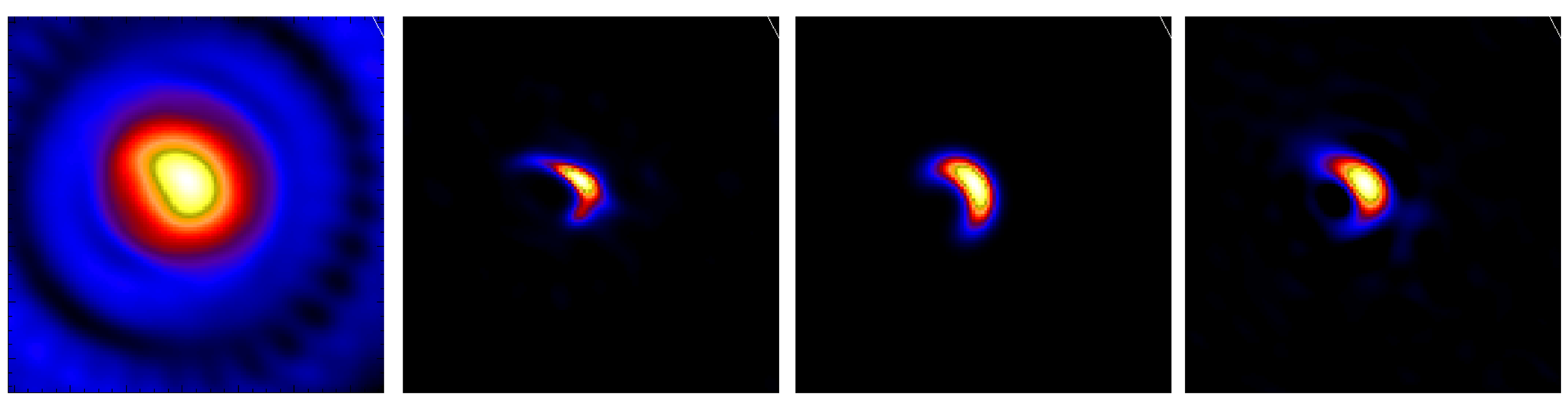
Application to real events

20 February 2002, 11:06:02 – 11:06:34 UT



back-projection MEM Forward fit uv - smooth

15 April 2002, 00:05:00 – 00:10:00



back-projection MEM Forward fit uv - smooth

References

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