Numerical assessment of the ROI CT problem in fan-beam geometries

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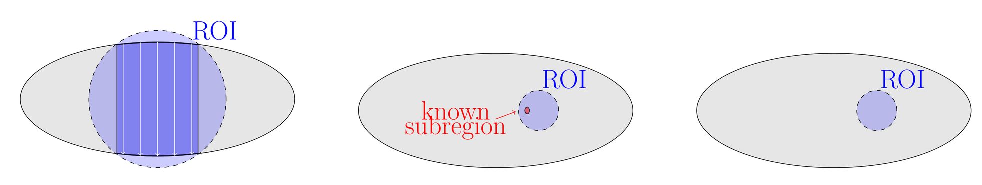
Motivations

Health hazards for patients due to ionizing radiations in Computed tomography (CT) can be reduced by limiting the irradiation to a subregion of the object to be reconstructed, the so-called *region-of-interest* (ROI) [1].

Goal

Obtaining a stable reconstruction of the ROI without any assumption on the size and location of the ROI and overcoming the ill-posedness of the problem and the instability of naive local reconstruction algorithms.

State of the art



Examples of recoverable regions:

(a) from (at least) one projection view the object is completely recovered;
(b) a known subregion inside the ROI is given;
(c) no assumption either on the size or on the location of the ROI, except for its convex shape.

2D problem setting

The aim of ROI CT is to reconstruct an integrable function f from its **Radon** projections y_0 known only within a subregion inside the field of view, while the rest of the image is ignored. This is accomplished by setting:

$$y_0(\theta,\tau) = M(\theta,\tau)\,\mathcal{R}f(\theta,\tau)$$

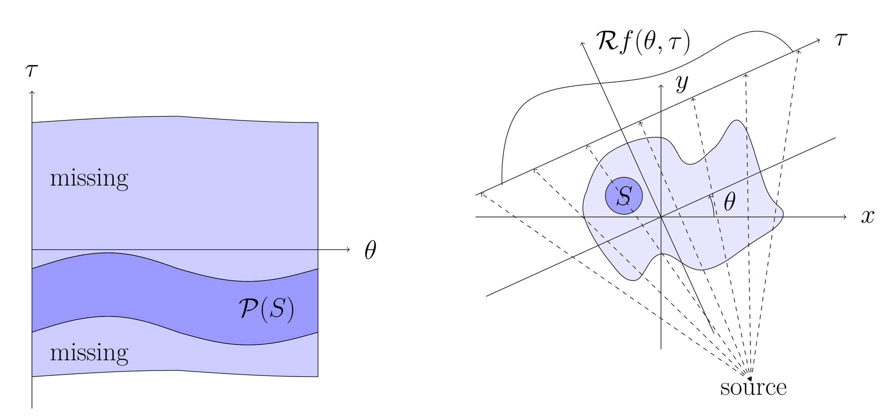
where

$$\mathcal{R}f(\theta,\tau) = \int_{\ell(\theta,\tau)} f(\mathbf{x})\,d\mathbf{x} = \int_{\mathbb{R}^2} \delta(\tau - \mathbf{x}\cdot e_\theta)\, f(\mathbf{x})\,d\mathbf{x}$$

is the *Radon transform* of f at (θ,τ) and the mask

$$M(\theta,\tau) = 1_{\mathcal{P}(S)}(\theta,\tau)$$

identifies the ROI S in the sinogram space [2].



Given y_0 defined on $\mathcal{P}(S)$, the goal is to extrapolate it to the region outside $\mathcal{P}(S)$, ensuring that the Radon projections $y = \mathcal{R}f$ comes from the Radon transform of a function $f \in L^1 \cap L^2$:

$$M\mathcal{R}f = My = y_0 \qquad \text{(data fidelity)}$$
$$(1-M)\mathcal{R}f = (1-M)y \qquad \text{(data consistency)}$$

2D discrete setting

Denoting with W the $KP \times N^2$ forward projection matrix, the data fidelity and consistency equations read as follows:

$$\mathbf{MWf} = \mathbf{My} = \mathbf{y}_0 \qquad \text{(data fidelity)}$$
$$(\mathbf{I}_{KP} - \mathbf{M})\,\mathbf{Wf} = (\mathbf{I}_{KP} - \mathbf{M})\,\mathbf{y} \qquad \text{(data consistency)}$$

where K = # projection angles, P = # detector elements, N = width in pixel of the reconstructed object.
Unfortunately, these equations alone do not lead to a unique solution [3]. A suitable one can be derived using a Tikhonov-like regularization:

$$\min_{\substack{f\in\Omega_f \\ y\ge 0}} \Psi(f,y)$$

where

$$\Psi(f,y) = \frac{1}{2}\,\|MWf - y_0\|_2^2 + \frac{1}{2}\,\|(\mathbf{I}_{KP} - M)(Wf - y)\|_2^2 + \lambda\,\|\Phi((\mathbf{I}_{KP} - M)y + y_0)\|_2^2$$

and Φ is the shearlet (resp. wavelet) transform [4].
Slight modifications of the objective function can be taken into account, coupling the regularization term with, for instance, a Total Variation term:

$$\widetilde{\Psi}(f,y) = \Psi(f,y) + \rho\,\mathrm{TV}_\delta(f)$$

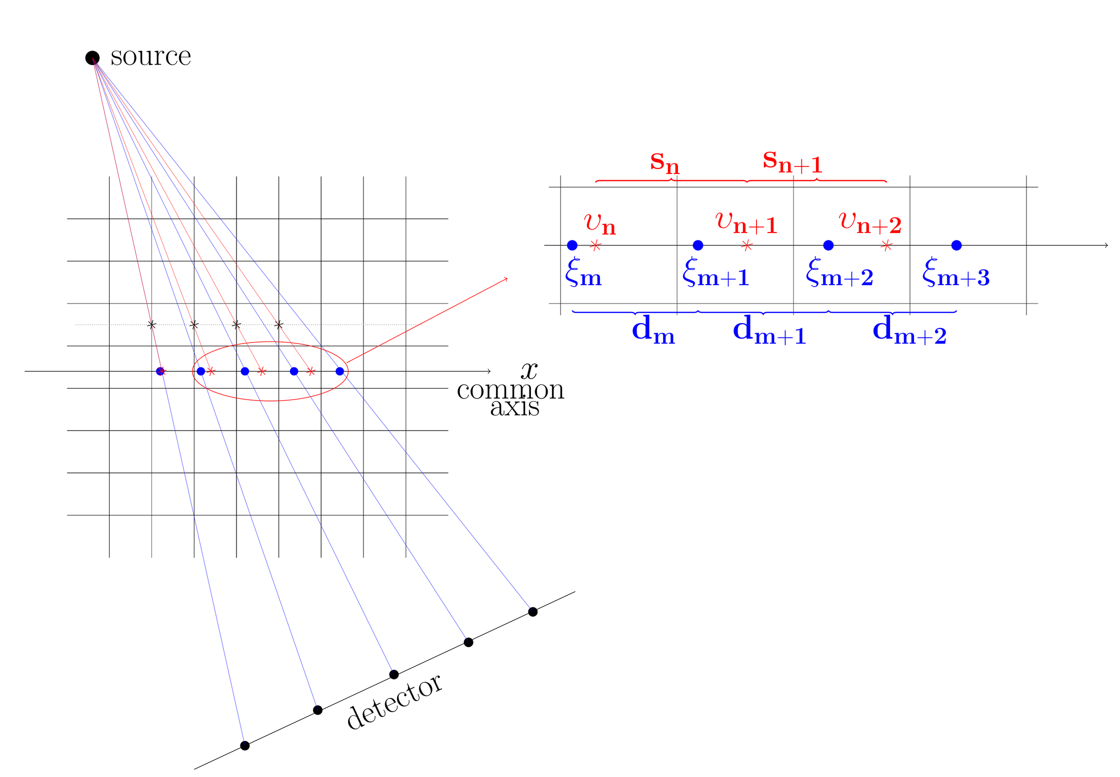
Here, λ and ρ are regularization parameters, δ is the TV smoothing parameter.

Distance-Driven method

Each object pixel (voxel) and detector cell is mapped onto a common axis (plane) by its projecting boundary midpoints.

$$s_n = \frac{\xi_{m+1} - \upsilon_n}{\upsilon_{n+1} - \upsilon_n} d_m + \frac{\upsilon_{n+1} - \xi_{m+1}}{\upsilon_{n+1} - \upsilon_n} d_{m+1}$$

The length of the overlap is used as projection weight [5].



- Low computational cost
- Avoids artifacts (*e.g.*, due to interpolation) of classical methods
- Allows for highly sequential memory access patterns.

Main Results

Objective function = Discrepancy terms + Reg. + TV

Radius	iter.	PSNR	iter.	Rel. Err.
0.5	1140	45.6933	1140	0.0249
	$\lambda = 5e-04$	$\rho = 0.1$	$\lambda = 5e-04$	$\rho = 0.1$
0.3	3168	40.3471	3168	0.10245
	$\lambda = 5e-04$	$\rho = 1$	$\lambda = 5e-04$	$\rho = 1$
0.25	2112	36.4519	2112	0.2005
	$\lambda = 5e-04$	$\rho = 1$	$\lambda = 5e-04$	$\rho = 1$
0.2	302	37.6381	302	0.24412
	$\lambda = 5e-04$	$\rho = 1$	$\lambda = 5e-04$	$\rho = 1$
0.15	604	34.6311	604	0.50634
	$\lambda = 5e-04$	$\rho = 1$	$\lambda = 5e-04$	$\rho = 1$
0.1	943	31.1664	943	1.2489
	$\lambda = 5e-04$	$\rho = 1$	$\lambda = 5e-04$	$\rho = 1$

Objective function = Discrepancy terms

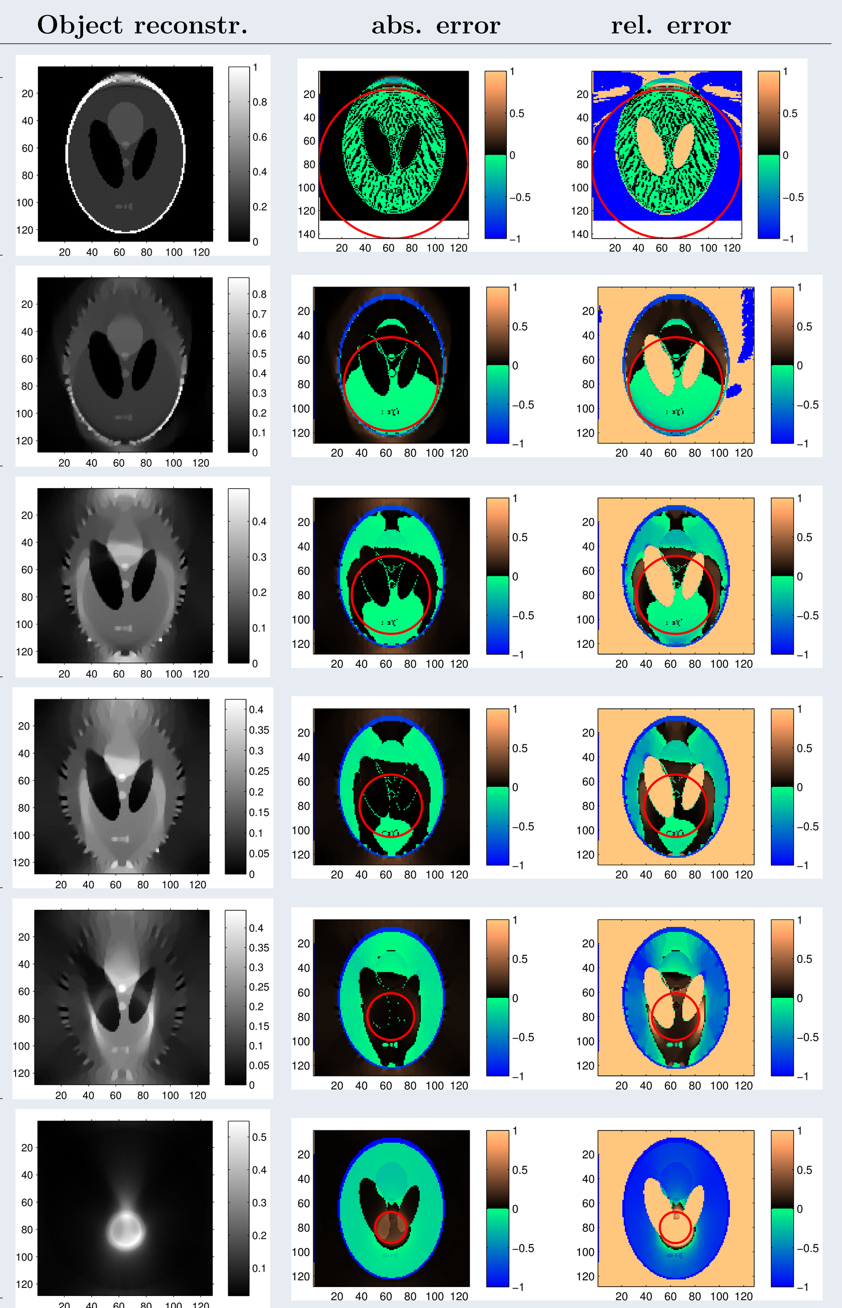
Radius	iter.	PSNR	iter.	Rel. Err.
0.5	7000	45.008	7000	0.026944
0.3	550	36.9959	550	0.15069
0.25	194	36.7668	194	0.19336
0.2	179	35.191	179	0.32356
0.15	7000	33.2901	7000	0.59087
0.1	2	35.9742	2	0.71803

Objective function = Discrepancy terms + Reg.

Radius	iter.	PSNR	iter.	Rel. Err.
0.5	1054	41.495	1054	0.040376
	$\lambda = 5e-04$		$\lambda = 5e-04$	
0.3	1495	34.8624	1495	0.19264
	$\lambda = 5e-04$		$\lambda = 5e-04$	
0.25	1259	31.9533	1259	0.33654
	$\lambda = 5e-04$		$\lambda = 5e-04$	
0.2	678	31.654	678	0.48619
	$\lambda = 5e-04$		$\lambda = 5e-04$	
0.15	2189	28.4602	2189	1.0304
	$\lambda = 5e-04$		$\lambda = 5e-04$	
0.1	2373	28.7556	2373	1.6484
	$\lambda = 5e-04$		$\lambda = 5e-04$	

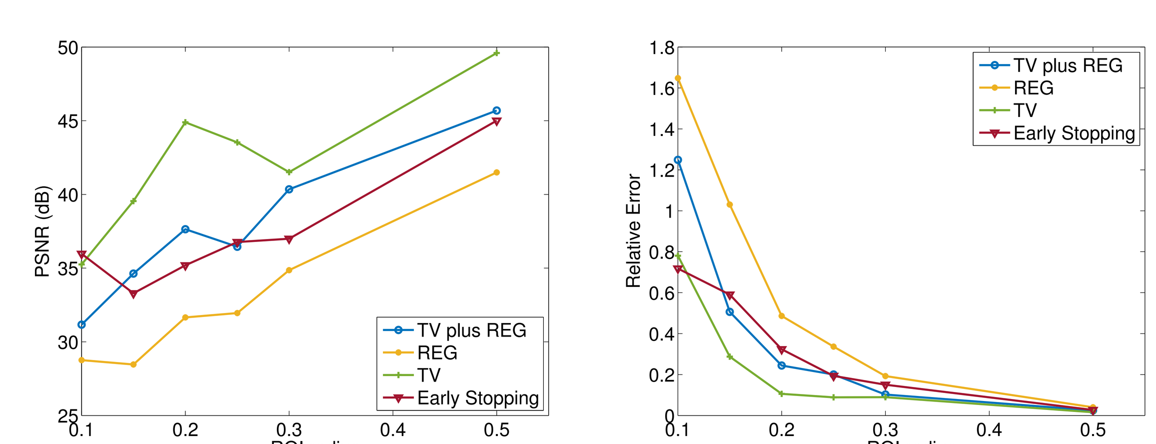
Optimal parameters values

Radius	iter.	PSNR	iter.	Rel. Err.
0.5	1642	49.5938	1642	0.015892
	SGP + TV, $\rho = 0.01$		SGP + TV, $\rho = 0.01$	
0.3	2933	41.5101	2933	0.089613
	SGP + TV, $\rho = 1$		SGP + TV, $\rho = 1$	
0.25	1075	43.5306	1075	0.088751
	SGP + TV, $\rho = 1$		SGP + TV, $\rho = 1$	
0.2	1910	44.8966	1910	0.10585
	SGP + TV, $\rho = 1$		SGP + TV, $\rho = 1$	
0.15	556	39.5493	556	0.28743
	SGP + TV, $\rho = 1$		SGP + TV, $\rho = 1$	
0.1	2	35.9742	2	0.71803
	SGP		SGP	



Scaled gradient projection method

For the solution an iterative approach based on the scaled gradient projection method (SGP) has been considered [6]. It is a first-order descent method for convex (and non-convex) functions, with adaptive step-length selection.



This method is particularly effective when the projection onto the feasible region is not a heavy task, such as in the case of box constraints possibly, coupled with a single linear constraint.

Future perspectives

- Investigate sparse reconstruction
- Obtain stable reconstructions from (Poisson) noisy sinogram
- Separable footprint method for system matrix
- Apply the same machinery to helical CT

References

[1] R. Clackdoyle and M. Defrise, *Tomographic Reconstruction in the 21st century. Region-of-interest reconstruction from incomplete data*, IEEE Signal Processing, **60** (2010).

[2] B. Goossens, D. Labate and B. Bodmann, *Region-of-interest Computed Tomography by Regularity-Inducing Convex Optimization* (2014).

[3] B. Vandeghinste, B. Goossens *et al.*, *Iterative CT reconstruction using shearlet-based regularization*, IEEE Trans. Nuclear Science **5** (2013).

[4] G. Kutyniok and D. Labate, *Shearlets. Multiscale Analysis for Multivariate Data*, Birkhäuser (2012).

[5] B. De Man and S. Basu, *Distance-driven projection and backprojection in three dimensions*, Physics in Medicine and Biology, **7** (2004).

[6] S. Bonettini, R. Zanella, L. Zanni, *A scaled gradient projection method for constrained image deblurring*, Inverse Problems **25** (2009), 015002.

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